ARTICLE

Utilizing data-based artificial intelligence to enable science-based models and dynamic feedback controllers to adapt to disasters

Jose B. Cruz, Jr.*[1]

[1]National Academy of Science and Technology, Engineering Sciences and Technology Division, Bicutan, Taguig City, Philippines

When disasters (such as floods, typhoons, and other natural calamities) occur, dynamic physical systems might change structurally, and the science based models of the systems should change also. If the new behaviors exhibit instability, it would be imperative that the new situations be brought to safe conditions as quickly as possible by means of appropriate feedback control. In these cases, use of artificial intelligence in controllers that do not utilize feedback control would lead to further human fatalities and damages to property.

Selected results from science-based models and control of dynamic physical systems are collected in this paper because the modeling and control methodology for these classes of problems could be implemented automatically. Commercial software systems are widely available for designing the corresponding dynamic controllers.

Major types of disaster situations could be anticipated and each situation could be associated with a predetermined science based model and corresponding dynamic feedback controller. A front-end data-based AI system could be designed to determine what disaster occurred. The identified situation could be compared with a catalogue list of predetermined disasters to decide which predetermined disaster situation is closest. For example, a Bi-directional Associative Memory artificial neural network might be used. The predetermined corresponding dynamic feedback controller would be switched in automatically.

*Corresponding author
Email Address: joe.cruz@icloud.com
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The stabilization challenges posed by the changed behaviors of a cyber physical system after the occurrence of a disaster strongly suggest that mechanisms for science based models and dynamic feedback controllers be imbedded in the system whereby the data based artificial intelligence system would be used to identify and classify the disaster and the feedback controller would automatically adapt to it.

KEYWORDS

Science based models, feedback control technology, game theory, integrating control technology with artificial intelligence

1. HISTORICAL BACKGROUND

In the opening paragraph of the first chapter of the third edition of a widely used textbook (Russell and Norvig 2010), *intelligence* is characterized: “We call ourselves Homo sapiens -- man the wise -- because our intelligence is so important to us. For thousands of years, we have tried to understand how we think; that is, how a mere handful of matter can perceive, understand, predict, and manipulate a world far larger and more complicated than itself.” Next *Artificial Intelligence* is characterized: Artificial Intelligence (AI) “attempts not just to understand but also to build intelligent entities … the name itself was coined in 1956.” (Russell and Norvig 2010).

The field of AI is so vast and so extensive that it is inevitable to have so many definitions of what AI is. Russell and Norvig organized various definitions into four categories: (1) Thinking humanly, (2) Acting humanly, (3) Thinking rationally, and (4) Acting rationally. In this fourth definition category, there is an agent or computer agent.

A computer agent "is just something that acts (*agent* comes from the Latin *agere*, to do) … operate autonomously, perceive their environment, persist over a prolonged time period, adapt to change, and create and pursue goals. A rational agent is one that acts so as to achieve the best outcome or, when there is uncertainty, the best expected outcome … the rational-agent approach has two advantages over the other approaches. First, it is more general than the 'laws of thought' approach because correct inference is just one of several possible mechanisms for achieving rationality. Second, it is more amenable to scientific development than are approaches based on human behavior or human thought. The standard of rationality is mathematically well defined and completely general and can be 'unpacked' to generate agent designs that provably achieve it. Human behavior, on the other hand, is well adapted for one specific environment and is defined by the sum total of all the things that humans do. This book therefore concentrates on general principles of rational agents and on components for constructing them …achieving perfect rationality—always doing the right thing—is not feasible in complicated environments. The computational demands are just too high … we will adopt the working hypothesis that perfect rationality is a good starting point for analysis. It simplifies the problem and provides the appropriate setting for most of the foundational material in the field… acting appropriately when there is not enough time to do all the computations one might like." (Russell and Norvig 2010).

Clearly, Russell and Norvig prefer the fourth category definition of AI. In their description of the historical development of AI, they point out that the rapid progress and achievements in AI during the last three decades have tended to have more science basis. "Recent years have seen a revolution in both the content and the methodology of work in artificial intelligence. It is now more common to build on existing theories than to propose brand-new ones, to base claims on rigorous theorems or hard experimental evidence rather than on intuition, and to show relevance to real-world applications rather than toy examples. AI was founded in part as a rebellion against the limitations of existing fields like control theory and statistics, but now it is embracing those fields. … "Initially, the mathematical tools of control theory were quite different from AI, but the fields are coming closer together." (Russell and Norvig 2010).

In this paper we adopt the definition category that an AI machine acts rationally. Rationality means that rules of logic are applied to the extent possible, but in complex situations, application of logic might be infeasible at best and a simpler alternative is used.

2. AGENTS

Agents are defined in both artificial intelligence and in control and game theories.

2.a Agents in artificial intelligence

In AI machines, an agent is an entity that produces actions (through physical actuators) based on environmental observations (through physical sensors). In spite of the trend towards more science base in AI and the acceptance of agents to be rational, much of recent AI research employs data collected about an agent, combined with massive and fast computation and networking. Nevertheless, agents in AI are supposed to be rational.

2.b. Agents in control and game theories

In a complex system, even when there is only one stakeholder, it is often useful to have multiple controllers, each one producing action based on local observations. These controllers are called agents (Cruz and Simaan 1999, Cruz et al. 2010). When there is only one agent the controller is called a centralized controller. In systems with multiple stakeholders, each stakeholder might have multiple agents. Each agent belongs to only one stakeholder. In every case, an agent applies an action to the system based on local or even system-wide observation. An agent in a control system or a complex multi stakeholder system operating autonomously has the same meaning as a rational agent in artificial intelligence.

3. DYNAMICS AND CONTROL OF CYBER PHYSICAL SYSTEMS

A significant area of application and future applications of AI is control of physical dynamic systems. In this paper we focus attention to physical systems after the occurrence of disasters, such as electrical energy systems after a transmission line is disabled for a variety of reasons, potential landslides after a long and heavy rainfall, and impact of a water dam failure on flooding are three examples of dynamic systems in emergency mode that are mathematically modeled based on physical principles. These systems are dynamic in the sense that the rate of change of state depends on the present state and on the present action. Such systems are mathematically modeled by differential equations. There is a highly developed body of knowledge called control theory. For example, see some textbooks (Astrom and Murray 2008, Ogata 2010, Perkins and Cruz 1969) and another highly developed body of knowledge called game theory. J. von Neuman and O. Morgenstern (von Neumann and Morgenstern 1944) established the field. J. Nash (Nash 1950) wrote a mathematics doctoral dissertation that was the basis for a Nobel Prize in economics. LS Shapley (Shapley 1953), created a game value now known as the Shapley value, another Noble Laureate. RP Isaacs (Isaacs 1955) is the originator of differential games. YC Ho (Ho 1970) clarified the connection between control and differential game theory. , CI Chen and JB Cruz (Chen and Cruz 1972) were the first to consider dynamic Stackelberg games. MA Simaan and JB Cruz (Simaan and Cruz 1973a, Simaan and Cruz 1973b) reframed dynamic Stackelberg game theory, providing mathematical proofs for various results and showed that the Stackelberg strategies do not necessarily satisfy Bellman's principle of optimality. This violation of the principle of optimality was renamed as time inconsistency in economic policy (Kydland and Prescott 1977), and later Kydland and Prescott won the Noble prize in economics in 2004 in part on the basis of their paper. MA Simaan and JB Cruz (Simaan and Cruz 1973c) extended the dynamic Stackelberg concept to many players. JB Cruz (Cruz 1975) introduced the dynamic Stackelberg concept to the economics community, and he extended the Stackelberg strategy to more than two levels (Cruz 1978). T Basar and Olsder (Basar and Olsder 1998) is a definitive resource book on dynamic game theory., Given sensor measurements in real time, the control problem is to determine the mathematical operations on the sensor measurement to produce "actuation" signals as input to the physical system, to steer it to a desired state. There are algorithms and software systems to do this. If there were multiple stakeholders in the systems, the needed computation would be guided by game theory.

The modeling and control of the physical system may be implemented through a wireless communication network connected to the actuator inputs and observation outputs of the physical system. The entire system is called a cyber-physical system (Rajkunar et al. 2010). When the automation can be accomplished with no human assistance, we have artificial intelligence.

In this section, we summarize some of the results in control theory for dynamic physical systems that involve science based modeling of the dynamic physical system, and the structure of the dynamic controller model, to provide a basis for enhancement of data-based artificial intelligence, in dealing with dynamic cyber physical systems. This enhancement is particularly appropriate after the occurrence of disasters, whereby the dynamics of the physical systems are altered as a consequence of the disasters. The new dynamics of the altered systems might be such that without appropriate feedback control the systems might be unstable causing further disasters. Direct application of artificial intelligence might not stabilize the dynamic systems. The results from control theory summarized in this section can be automated and artificial intelligence methods could be applied to the augmented dynamic models to further improve the operation of the systems.

3.1 Second-order physical system with control, example 1

As an illustrative example, suppose that a space satellite for communications just lost its motion control. In normal operation it points to a specific point on earth where there is a receiving station. Let us consider its motion along one dimension. It is similar to a rotating shaft whose angular position in radians is denoted by x_1, its angular velocity in radians per second by x_2. From elementary physics, assuming that the shaft rotates on a frictionless support we have

$$\frac{dx_1}{dt} = x_2$$

$$J\frac{dx_2}{dt} = \tau \tag{a}$$

where J is the angular moment of inertia and τ is the torque on the shaft. The angular position and the angular velocity are called the two state variables of the mechanical system. To simplify the notation assume a normalized value of J as 1. Consider the torque as a control input. Another possible example might be the position of an optical rotating disc information storage, and we need to position it precisely at a specified value. We can find an exact torque-time profile in a short time duration to do this but in practice it would not work because the torque can not be implemented numerically exactly. The disc will keep rotating with no torque applied. This is an example of open-loop control where the input torque is a function of time and not dependent on the angular position or angular velocity. Integrating the derivative of the angular velocity equation,

$$x_2(t) - x_2(0) = \int_0^t \tau\, d\xi \tag{b}$$

For specificity, suppose that $x_2(0) = \alpha_2 = 2$. Consider choosing

$\tau = \beta$ for $0 \le t \le, \delta$, (c)

$\tau = -\beta$ for $\delta \le t \le 1$ (d)

and

$\tau = 0$ for $t > 1$. (e)

Thus,

$$x_2(t) = 2 + \beta t \text{ for } 0 \le t \le \delta \tag{f}$$

$$x_2(t) = 2 + 2\beta\delta - \beta t \text{ for } \delta \le t \le 1 \tag{g}$$

$$x_2(t) = 2 - \beta(1 - 2\delta) \text{ for } t > 1 \tag{h}$$

Integrating the angular velocity equation $x_2(t)$

$$x_1(t) - x_1(0) = \int_0^t x_2(\xi)d\xi \tag{i}$$

Suppose that $x_1(0) = \alpha_1 = 3$. For $0 \le t \le \delta$

$$x_1(t) = 3 + 2t + \beta t \tag{j}$$

For $\delta \le t \le 1$

$$x_1(t) = 3 + \int_0^\delta [2 + \beta\xi]d\xi + \int_\delta^t [2 + 2\beta\delta - \beta\xi]d\xi \tag{k}$$

$$x_1(t) = 3 + 2t - \beta\delta^2 + 2\beta\delta t - \tfrac{1}{2}\beta t^2 \tag{l}$$

For $t > 1$,

$$x_1(t) = 5 - \beta(\delta^2 - 2\delta + \tfrac{1}{2}) \tag{m}$$

At $t = 1$,

$$x_1(1) = 5 - \beta(\delta^2 - 2\delta + \tfrac{1}{2}) \tag{n}$$

Suppose that the desired value for the angular position at t = 1 is $x_1(1) = 1$. Then

$$\delta^2 - 2\delta + \frac{1}{2} = \frac{4}{\beta} \tag{o}$$

The angular velocity at t = 1 is

$x_2(1) = 2 + 2\beta\delta - \beta$, from Eq. (h)

Since we want to stop the motion at $t = 1, x_2(1) = 0$

$$1 - 2\delta = \frac{2}{\beta} \tag{p}$$

Substituting Eq. (p) in Eq. (o),

$$\delta^2 - 2\delta + \frac{1}{2} = 2(1 - 2\delta) \tag{q}$$

$$\delta^2 + 2\delta - \frac{3}{2} = 0 \tag{r}$$

$$\delta = -1 + \sqrt{\tfrac{10}{4}} \approx 0.5811 \tag{s}$$

$$\beta = \frac{2}{1-2(-1+\sqrt{10/4}} = \frac{2}{3-\sqrt{10}} \approx -12.325 \tag{t}$$

From the foregoing, there is only one feasible solution for $\delta, 0 \le \delta \le 1$. If the torque time profile can be implemented exactly, then the space object will stop tumbling at time t = 1 at the desired specified position, and it will remain there for t > 1. Unfortunately, the actual hardware and software will always involve numerical inaccuracies. So that $x_2(1) \neq 0$. Thus, $x_2(t) \neq 0$ and $x_1(t) \neq 1$, for $t \ge 1$. The space object will keep tumbling.

An AI machine that does not utilize appropriate feedback will surely fail.

Next, suppose that we make the torque proportional to the negative of the angular position by using a position sensor so that $\tau = kx$. Substituting this in the equations for the shaft we have

$$\frac{dx_2}{dt} = kx_1 \tag{u}$$

Taking the derivative of the above equation, we have

$$\frac{d^2x_2}{dt^2} = k\frac{dx_1}{dt} = kx_2 \tag{v}$$

where k is the proportionality constant of the sensor. From elementary physics and differential equation theory, the characteristic equation is

$$s^2 - k = 0 \tag{w}$$

If k were chosen to be positive, the angular position would keep exponentially increasing. If k is chosen to be negative, the angular position would oscillate at a frequency that is the square root of - k. This is not an appropriate feedback control.

Next, suppose that in addition to a sensor for the angular position we have a sensor for the angular velocity also, and suppose we construct the total torque as a linear combination of a constant external torque, and the outputs from the two sensors with proportionality constants,

$$\frac{dx_1}{dt} = x_2 \tag{1}$$

$$\frac{dx_2}{dt} = k + k_1 x_1 + k_2 x_2 \tag{2}$$

The second order differential equation for x_1 from Eqs. (1) and (2) is

$$\frac{d^2x_1}{dt^2} - k_2 \frac{dx_1}{dt} - k_1 x_1 = k \tag{3}$$

From elementary differential equation theory, the characteristic equation for Equations (1) and (2) is

$$s^2 - k_2 s - k_1 = 0$$

To guarantee stability, the two parameters in the characteristic equation need to have negative values. Depending on the values of these proportionality constants, the angular position would have exponentially damped time trajectories or exponentially damped sinusoids. Since the two proportionality constants are design parameters, it is clear that any characteristic root (complex conjugate pair) could be realized by appropriate choices of k_1 and k_2.

Suppose that $x_1(0) = 5, x_2(0) = 3, x_1(1) = 1$. Let us choose $k_1 = -200, k_2 = -30$. The characteristic equation is $s^2 + 30s + 200 = 0$. The characteristic roots or eigenvalues are $s_1 = -10, s_2 = -20$.

$$x_1(t) = ce^{-10t} + be^{-20t} + a, x_2(t) = -10ce^{-10t} - 20be^{-20t}$$
$$x_1(0) = 5 = c + b + a,\ x_2(0) = 3 = -10c - 20b$$
$$c + b = 5 - a,\ c + 2b = -\frac{3}{10}$$
$$b = a - \frac{53}{10},\ c = \frac{103}{10} - 2a$$
$$1 = ce^{-10} + be^{-20} + a$$
$$1 = e^{10}(\frac{103}{10} - 2a) + e^{-20}(a - \frac{53}{10}) + a$$
$$a = \frac{1 - 10.3e^{-10} + 5.3e^{-20}}{1 + e^{-20} - 2e^{-10}} \approx 1$$
$$b = a - 5.3 \approx -4.3$$
$$c = 10.3 - 2a \approx 8.3$$
For $t \geq 1$
$$x_1(t) \approx 1$$
$$x_2(t) \approx 0$$

The external constant torque in Eq. (3) is obtained from

$$\frac{k}{-k_1} = a, \text{ or}$$

$$k = -ak_1 = 200[\frac{1 - 10.3e^{-10} + 5.3e^{-20}}{1 + e^{-20} - 2e^{-10}}]$$

Even if there is a reasonable error in setting the values of the proportionality constants and all the initial conditions, the space object will settle to an angular position close to 1.0. This feedback controller will stabilize the system.

In this illustrative example, there is a stabilization issue, and as indicated above, feedback control theory demonstrates that the system can be stabilized. Note that a physics-based model of the shaft as a second-order differential equation was introduced. Furthermore, sensors were introduced to measure the two state variables of the system. The actuator is assumed to still function. If the torque were to be determined as an independent forcing function experimentally, based on massive data of previous torque inputs, with no feedback control, the system might become useless if the torque fails to stabilize the system. This illustrative example shows the importance of incorporating a science-based model of the physical system and utilizing control theory to establish the infrastructure of the controller, when the physical system is dynamic. Thereafter, data-based AI may be used to refine the operation.

When the order of a dynamic physical system is high, it is advantageous to denote the state variables as components of a vector and to process the multiple model equations using matrix algebra. For example, the two equations might be written in matrix-vector form as

$$\frac{dx}{dt} = Ax + Bu \tag{4}$$

where

$$x=\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, A=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B=\begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (5)$$

and u is a control vector. In this illustrative example u is the one-dimensional torque τ.

3.2 General n[th] order linear time-invariant system

The standard notation for this class of systems is given by

$$\frac{dx}{dt}=Ax+Bu \quad (6)$$

$$y=Cx \quad (7)$$

where x is an n-dimensional state vector, u is the m-dimensional control vector, and y is the p-dimensional output vector. A, B, and C are matrices with dimensions nxn, nxm, and pxn respectively. The components of the output vector are the measured quantities of the physical system. Equations (6) and (7) provide the science-based model of a general n[th] order linear time-invariant physical system (Ogata 2010).

3.3 Full state vector available as output

Suppose that the entire state vector could be measured so that

$$y=x \quad (8)$$

If a linear feedback control vector (called linear state feedback control)

$$u=Kx \quad (9)$$

is applied to the system, Equation (6) becomes

$$\frac{dx}{dt}=(A+BK)x \quad (10)$$

Without state feedback control, the characteristic roots are the eigenvalues of the matrix *A*. With state feedback the characteristic roots are the eigenvalues of the matrix (*A* + *BK*). When the system in Equation (6) is controllable, the elements of the mxn control matrix K can be chosen such that any specified set of eigenvalues of (*A* + *BK*) can be realized. Thus, this feedback controller can completely change the dynamics of the system by moving any and all eigenvalues to any desired set of values. A necessary and sufficient condition for the system to be "controllable" is that the rank of the controllability matrix

$$\Psi=\begin{bmatrix} B & AB & \dots & (A)^{n-1}B \end{bmatrix} \quad (11)$$

is n (Kalman, Ho, and Narendra 1963).

3.4 Illustrative second order physical system with full state feedback control, example 2

In the second-order system of Example 1 in Section 2.2, the control matrix K is a two-dimensional row vector

$$K=\begin{bmatrix} k_1 & k_2 \end{bmatrix} \quad (12)$$

The A and B matrices are defined in Equation (5). Thus

$$A+BK=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}+\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} k_1 & k_2 \end{bmatrix}=\begin{bmatrix} 0 & 1 \\ k_1 & k_2 \end{bmatrix} \quad (13)$$

and the eigenvalues of

$$\begin{bmatrix} 0 & 1 \\ k_1 & k_2 \end{bmatrix}$$

are the roots of the characteristic equation given by Equation (3). The two control parameters can clearly be chosen to obtain any desired set of two eigenvalues, provided that complex eigenvalues must be a complex conjugate pair. The eigenvalues will have negative real parts (needed for stability) provided that the control parameters are negative numbers (negative feedback).

3.5 Estimating the state vector using an observer

Depending on the application, it might not be convenient or even possible to measure all n components of the state vector. Instead the measurement output vector is a linear combination of the state variables as given in Equation (7) where the dimension p of the output vector is less than or equal to the dimension n of the state vector. Under certain conditions on the matrices *A* and *C,* it is possible to construct a dynamic system, as part of the controller system such that the state of the additional dynamic system becomes asymptotically equal to the state of the original system. The asymptotic convergence of the state of the additional system to the state of the original system can be made arbitrarily fast. The new dynamic system is called a state observer (Luenberger 1966). The observer is modeled by the following vector equation:

$$\frac{d\hat{x}}{dt}=A\hat{x}+Bu-\hat{K}(y-C\hat{x})=(A+\hat{K}C)\hat{x}+\begin{bmatrix} B & -\hat{K} \end{bmatrix}\begin{bmatrix} u \\ y \end{bmatrix} \quad (14)$$

$$\hat{y}=\hat{x} \quad (15)$$

where $\hat{x}$ is the n-dimensional state vector of the observer, $\hat{y}$ is the output vector of the observer and it is equal to the state vector of the observer, Equation (14) may be rewritten in standard form as

$$\frac{d\hat{x}}{dt}=\hat{A}\hat{x}+\hat{B}\hat{u} \quad (16)$$

where

$$\hat{A}=A+\hat{K}C \quad (17)$$

$$\hat{B}=\begin{bmatrix} B & -\hat{K} \end{bmatrix} \quad (18)$$

and

$$\hat{u} = \begin{bmatrix} u \\ y \end{bmatrix} \quad (19)$$

The observer input vector is a composite of vectors u and y

The control input of the original dynamic system is chosen as

$$u = K\hat{x} \quad (20)$$

The original system plus the controller that contains an observer is of order 2n. The entire dynamic system may be written as

$$\frac{d}{dt}\begin{bmatrix} x \\ \hat{x} \end{bmatrix} = \begin{bmatrix} A & BK \\ -\hat{K}C & A+BK+\hat{K}C \end{bmatrix}\begin{bmatrix} x \\ \hat{x} \end{bmatrix} \quad (21)$$

The eigenvalues of the 2nth order system are the eigenvalues of the matrix

$$\begin{bmatrix} A & BK \\ -\hat{K}C & A+BK+\hat{K}C \end{bmatrix}$$

To provide a better insight as to where these eigenvalues are, let us transform our 2n-dimensional state vector. Denote the difference $(x - \hat{x})$ $by\ e$

$$e = (x - \hat{x}) \quad (22)$$

Let us transform the state vector in Equation (21) to

$$\begin{bmatrix} x \\ \hat{x} \end{bmatrix} = \begin{bmatrix} I & 0 \\ I & -I \end{bmatrix}\begin{bmatrix} x \\ e \end{bmatrix} \quad (23)$$

where I is the n-dimensional identity matrix and 0 is a nxn zero matrix. So, Equation (21) can be rewritten as

$$\frac{d}{dt}\begin{bmatrix} x \\ e \end{bmatrix} = \begin{bmatrix} I & 0 \\ I & -I \end{bmatrix}^{-1}\begin{bmatrix} A & BK \\ -\hat{K}C & A+BK+\hat{K}C \end{bmatrix}\begin{bmatrix} I & 0 \\ I & -I \end{bmatrix}\begin{bmatrix} x \\ e \end{bmatrix} \quad (24)$$

Since

$$\begin{bmatrix} I & 0 \\ I & -I \end{bmatrix}^{-1} = \begin{bmatrix} I & 0 \\ I & -I \end{bmatrix}$$

Equation (24) becomes

$$\frac{d}{dt}\begin{bmatrix} x \\ e \end{bmatrix} = \begin{bmatrix} I & 0 \\ I & -I \end{bmatrix}\begin{bmatrix} A & BK \\ -\hat{K}C & A+BK+\hat{K}C \end{bmatrix}\begin{bmatrix} I & 0 \\ I & -I \end{bmatrix}\begin{bmatrix} x \\ e \end{bmatrix} \quad (25)$$

or

$$\frac{d}{dt}\begin{bmatrix} x \\ e \end{bmatrix} = \begin{bmatrix} A+BK & -BK \\ 0 & A+\hat{K}C \end{bmatrix}\begin{bmatrix} x \\ e \end{bmatrix} \quad (26)$$

The matrix

$$\begin{bmatrix} A+BK & -BK \\ 0 & A+\hat{K}C \end{bmatrix}$$

is block-triangular, so that its 2n eigenvalues are the n eigenvalues of $(A + BK)$ and the n eigenvalues of $(A + \hat{K}C)$

If the elements of the observer gain matrix $\hat{K}$ can be chosen so that the all eigenvalues of $(A + \hat{K}C)$ can have negative real parts and the magnitudes of the real parts are large, then the time trajectories of the components of the vector e will exponentially tend to zero very quickly and that the state vector of the observer will tend to the state vector of the original system in Equation (6) very quickly. This is the case when the system is "observable". A necessary and sufficient condition for the system to be observable is that the rank of the observability matrix

$$\Omega = \begin{bmatrix} C' & A'C' & \ldots & (A')^{n-1}C' \end{bmatrix}' \quad (27)$$

is n, where (') denotes transpose of a matrix.

The observer is part of the controller and the controller is an nth order dynamic system, Controller signals might be generated remotely through a wireless communication network and the resulting cyber physical system would be 2nth order system. Some of the state variables might be computable from the output y and so it is not necessary to estimate them. There are methods for obtaining reduced order observers (Ogata 2010).

The complete dynamic controller can be constructed using Equations (14) and (20), where y is the measured (available) output of the system to be controlled. Alternatively, we can substitute Equation (20) into Equation (14) and obtain

$$\frac{d\hat{x}}{dt} = (A + BK + \hat{K}C)\hat{x} - \hat{K}y \quad (28)$$

The design parameters are the elements of the K and $\hat{K}$ matrices.

3.6 Illustrative second-order system with state observer in the controller, example 3

Suppose that the output in Illustrative Example 1 is only the shaft angular position as measured by one sensor and no other measurements are available. Let us design an observer as part of the controller for estimating the state vector. The A and B matrices are given in Equations (4) and (5). The C matrix is given by

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad (29)$$

Denote the observer gain matrix by

$$\hat{K} = \begin{bmatrix} \hat{k}_1 \\ \hat{k}_2 \end{bmatrix} \quad (30)$$

The controller gain matrix is given in Equation (12). From the general observer model of Equation (28), and using Equations (5), (12), (29), and (30), the general observer equation (28) becomes

$$\frac{d\hat{x}_1}{dt} = \hat{k}_1\hat{x}_1 + \hat{x}_2 - \hat{k}_1 y \tag{31}$$

$$\frac{d\hat{x}_2}{dt} = (k_1 + \hat{k}_2)\hat{x}_1 + k_2\hat{x}_2 - \hat{k}_2 y \tag{32}$$

and the control in Equation (20) becomes

$$u = k_1\hat{x}_1 + k_2\hat{x}_2 \tag{33}$$

The eigenvalues of the entire fourth order system are the roots of

$$s^2 - \hat{k}_1 s - \hat{k}_2 = 0 \tag{34}$$

and the roots of Equation (3). By choosing both observer parameters as negative numbers (negative feedback), the observer state vector will exponentially converge to the original system state vector. Increasing the absolute value of $\hat{k}_1$ will increase the speed of convergence provided that the absolute value of $\hat{k}_2$ is also relatively large also.

The controller-observer dynamic system can be constructed using the model in Equations (31), (32) and (33). As stated earlier, we can use the output, which is equal to the first state variable of the system, instead of the first state variable of the observer. Furthermore, we can construct a first order observer. However, the purpose here is to indicate that the entire state can be estimated through an observer.

3.7 Obtaining the controller matrix through optimal control theory

One of the most widely used parts of optimal control theory pertains to regulator theory using state space representation of dynamic systems. The development of this theory began in the late 1950s (Kalman 1960a, Kalman 1964). Regulator theory deals with developing process mechanisms so that certain system variables maintain constant values or return to the specified constant values relatively quickly. Much of industrial applications of control theory deal with regulation. The variables to be regulated could be physical variables such as temperature, pressure, voltage frequency, thickness of materials, angular position of space antennas, etc. Without loss of generality the theory is developed for zero desired values. The starting point of optimal control theory is the specification of a performance criterion of a quantity to be minimized. For example, consider the scalar functional

$$J = \int_0^\infty (x'Qx + u'Ru)dt \tag{35}$$

The vectors x and u are the state and control vectors and the time evolution of the state is given. In the linear case the dynamics are given in Equation (6) R and Q are constant positive definite matrices. The optimal control problem is to determine an optimal control u to minimize J. Intuitively, when the initial state at time zero is not zero, we need to find a control time trajectory so that the sum of the energy expended in control plus the quadratic error in regulation is as small as possible. The developed theory states that the optimal control is (Kalman 1960a, Kalman 1964)

$$u = -R^{-1}B'Px \tag{36}$$

where P is the symmetric nxn positive definite matrix solution to the algebraic matrix Riccati equation

$$PA + A'P - PBR^{-1}B'P + Q = 0 \tag{37}$$

The theory guarantees the existence of a positive definite solution. Furthermore, the theory guarantees that all the eigenvalues of the matrix

$$A - BR^{-1}B'P$$

of the state feedback system have negative real parts, and hence the feedback system is stable. Commercial software is available for solving Equation (37). Comparing Equations (9) and (36), if we use

$$K = -R^{-1}B'P \tag{38}$$

stability would be guaranteed. Thus, regardless of specific interest in optimal regulator theory, Equations (37) and (38) may be used to design K, with Q and R regarded as design parameters.

3.8 Estimating the state vector through Kalman filtering theory

In the theory of Kaman filters (Kalman 1960b, Kalman and Bucy, 1961) for continuous time, the dynamic model in Equation (6) is modified to include process noise

$$\frac{dx}{dt} = Ax + Bu + w \tag{39}$$

where w is noise vector that is assumed to be "white". It has zero mean and a covariance matrix

$$E[\{w(t)w'(\tau)\} = Q_e\delta(t-\tau) \tag{40}$$

where Q_e is a symmetric positive definite matrix and

$$\delta(t-\tau)$$

is the Dirac delta "function" that is zero everywhere except at

$$t - \tau = 0$$

where it is "infinite", and its integral is equal to one. We can approximate it by a very tall rectangular pulse with width Δ and height $1/_\Delta$ and Δ is arbitrarily small. The Fourier transform of the Dirac delta "function" is a constant. White noise has a flat frequency spectrum.

Similarly, the output description given in Equation (7) is replaced by

$$y = Cx + v \tag{41}$$

where v is white measurement noise, and it has zero mean and a covariance matrix

$$E\{v(t)v'(\tau)\}=R_e\delta(t-\tau) \tag{42}$$

where R_e is a symmetric positive definite matrix. It is desired to estimate the state vector $x(t)$ by a vector estimator denoted by $\hat{x}(t)$ such that the statistical expectation

$$J_e=E\{[x(t)-\hat{x}(t)]'[x(t)-\hat{x}(t)]\} \tag{43}$$

is minimized. From Kalman Filtering theory (Kalman 1960b, Kalman and Bucy 1961) the answer is the conditional expectation of $x(t)$ given the data $y(t)$ up to time t,

$$\hat{x}(t)=E\{x(t)|\,y(t)\} \tag{44}$$

The conditional expectation can be calculated from the following equations

$$\frac{d\hat{x}}{dt}=(A+BK+K_eC)\hat{x}-K_ey \tag{45}$$

where

$$K_e=-P_eC'R_e^{-1} \tag{46}$$

and P_e is the symmetric positive definite solution of the algebraic matrix Riccati equation

$$AP_e+P_eA'-P_eC'R_e^{-1}CP_e+Q_e=0 \tag{47}$$

The Kalman Filtering theory guarantees the existence of a symmetric positive definite solution of Equation (47). Furthermore, the eigenvalues of $(A+\widehat{K}C)$ are guaranteed to have negative real parts.

The structure of the state observer discussed earlier is identical to the structure of the Kalman Filter. Although the Kalmar Filter provides the optimum estimate of the state under noisy conditions, the methodology for calculating the Kalman Filter gain matrix may be utilized for obtaining a stable dynamic estimator.

3.9 Multi-Criteria Optimal Control Theory

In realistic applications, in addition to stabilization, there might be multiple attributes desired. For example cost, latency, environmental impact, reliability, sustainability, and safety are prime examples of attributes that could be translated into multiple criteria. Suppose that for a given controller design, these criteria are normalized and calculated. Suppose that several (finite number) tentative designs are created as final candidates for choosing the final design (For example for different choices of the matrices Q and R). A weighted sum of the normalized criteria for each tentative design is calculated. Now choose the design with the highest weighted sum. The weights to choose are of course subjective, but a default choice of weights can be incorporated if the process is completely automated. The final choice is Pareto optimal. Furthermore, there are subspaces of the weights such that the final choice remains the same (Cruz and Almario 2018).

4. AUTOMATION OF FEEDBACK CONTROL OF CYBER PHYSICAL SYSTEMS

In Section 3, we described a selection of feedback control technology appropriate for controlling cyber physical systems that can be adequately represented by science based linear dynamic models. Given the numerical values of elements of matrices A, B, C, Q, R, Q_e, R_e, the numerical values of the elements of the controller gain matrix K and the numerical values of the elements of the Kalman Filter gain matrix $\widehat{K}$ can be computed automatically using commercially available software. Specifically, the algebraic matrix Riccati equations can be solved using a MATLAB Tool Box.

The numerical values for various model matrices for a specific cyber physical system may be obtained offline before the system becomes operational. Hence, the feedback controller can be designed and installed offline. In addition, alternative sets of numerical values of the model matrices may be stored. For example, these could be small variations from the values obtained earlier. The corresponding controller matrices may be calculated for each of the sets of model matrices. During operation, the controller may be switched to correspond to any of the stored model values. If the system had been in operation for a period of time, it would be conceivable that one of the alternate controllers could provide a better fit.

5. FRAMEWORK FOR UTILIZING ARTIFICIAL INTELLIGENCE TO ENABLE SCIENCE BASED MODELS AND DYNAMIC FEEDBACK CONTROLLERS TO ADAPT TO DISASTERS

When disasters occur, dynamic physical system might undergo drastic changes in dynamic behavior. Perhaps the existing controller could not adequately control the surviving portion of the system. If we could have a model of the changed system, then perhaps we could change the controller quickly. So, what might be a good course of action?

A suggested framework for using science-based modeling and feedback control technology, and data based artificial intelligence is described below:

(a) Develop several critical scenarios that are most likely, after a disaster. This is accomplished offline. For each scenario create a science-based model, complete with numerical values for the system. Create a catalogue of dynamic models for each possible scenario. More scenarios could be added to enhance the effectiveness of the entire process.

(b) For each model in (a) create the associated feedback control as summarized in section 4. Expand the catalogue in (a) to include the associated feedback controller.

(c) Create a data-based AI system to examine data on line, after the occurrence of a disaster, to identify and classify the disaster scenario, by comparing with the catalogue created in (a), to decide which predetermined disaster situation is closest. Suppose that there is an emergency event and that as a consequence of the event, the dynamics of the physical system change. In the simplest case suppose that the order of the system remains the same, but the numerical values of the elements of A, B, and C are changed. The numerical values of the control matrix gain and the observer matrix gain were calculated for the old values of the A, B, and C matrices of the dynamic system before the disastrous event occurred.

Specific scenarios of disasters could be anticipated, and the new A, B, and C matrices could be predetermined for each disaster category. Then AI might be deployed to detect and identify the type of disaster that occurred and switch the appropriate controller and observer gains. For example, a Bi-directional Associative Memory artificial neural network might be used (Wang eta al 1990a, Wang et al 1990b, Wang et al 1991, Wang et al 1993). Data-based artificial intelligence methods such as deep learning (LeCun et al 2015, Schmidhuber et al 2015) might be utilized. Deep learning is a type of machine learning (Schmidhuber et al. 2015) and they are multilayer artificial neural networks. Artificial neural networks started with single layer versions in the 1980s and 1990s; see for example (Wang et al. 1990a, Wang et al. 1990b, Wang et al. 1991, Wang et al. 1993).

(d) Based on the result in (c), the predetermined corresponding dynamic feedback controller would be switched in automatically.

Developing the Science Base for Multiple Stakeholders

There is a body of knowledge called *game theory* that pertains to cyber physical systems with multiple stakeholders. In particular, when the system is sufficiently modeled by a linear system, quadratic performance criteria, and for a variety of solution concepts there are explicit results from game theory. There are matrix Riccati equations to be solved but these are more complicated than the ones for the theory when there is only one stakeholder. The various controllers for the different stakeholders need further study for automation. As in the simpler control case, AI might be deployed to detect the type of disaster that occurred so that an appropriate predetermined set of controllers may be implemented after the occurrence of a disaster. When the dynamics of the system change, as a consequence of a disaster, to become unstable with no feedback control, it is imperative that the various stakeholder controllers apply appropriate feedback to avert unsafe conditions. The suggested framework would be the same as in the single stakeholder case. That is, use AI to identify and classify the disaster scenario, and then use an automate feedback controller for each stakeholder.

6. SUMMARY AND CONCLUSIONS

When disasters occur, dynamic physical systems might change structurally, and the science-based models of the systems should change also. If the remnant physical systems were unstable, it would be imperative that the new situations are brought to safe conditions as quickly as possible. If AI were to be deployed with no feedback control there would be further disasters.

In this paper we proposed a new framework for utilizing data based artificial intelligence to enable science-based models and dynamic feedback controllers to adapt to disasters.

The framework includes the development of several critical scenarios that are most likely, after a disaster. This is accomplished offline. For each scenario a science-based model is to be created, complete with numerical values for the system. A catalogue of dynamic models for each possible scenario is to be created. More scenarios could be added to enhance the effectiveness of the entire process.

Proposed is creation of a data-based AI system to examine data on line, after the occurrence of a disaster, to identify and classify the disaster scenario, by comparing with a catalogue previously created, to decide which predetermined disaster situation is closest.

Some results from control theory that could be automated were collected. Starting from the development of science based dynamic models of linear cyber physical systems, to the development of the controller structure including estimation of the system state vector through a state observer or Kalman filter, explicit procedures were summarized for computing parameters of the dynamic controller. There are commercial software systems for these computations. Thus, the construction of these dynamic controllers can be automated.

The last stage in this framework is to switch in the feedback control associated with the post-disaster model identified by the AI classifier.

There is great merit in imbedding control theory, and game theory, into the AI framework, whenever dynamic cyber physical systems are involved.

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